The 3-Level HLM Model

James H. Steiger

Department of Psychology and Human Development
Vanderbilt University

Multilevel Regression Modeling, 2009

The 3-Level HLM Model

Introduction

We examine the 3-level HLM model and the introductory example in the HLM manual.

Basic Model Characteristics

The 3-level model extends the ideas from the 2-level model to a third level. This opens up many possibilities. For example, students can be nested within classrooms, which are in turn nested within schools. In that case, there are n_{jk} students nested within each of $j = 1, \ldots, J_k$ classrooms, in turn nested within each of $k = 1, \ldots, K$ schools.

Level-1 Model

At level 1, the outcome Y_{ijk} for case i within level-2 unit j and level-3 unit k is represented as

$$Y_{ijk} = \pi_{0jk} + \sum_{p=1}^{P} \pi_{pjk}\, a_{pjk} + e_{ijk} \tag{1}$$

The π_{pjk} are level-1 coefficients, with the corresponding a's the level-1 predictors. e_{ijk} is the level-1 random effect, with the assumption that

$$e_{ijk} \sim \mathrm{N}(0, \sigma^2) \tag{2}$$

Level-2 Model

At level 2, the π coefficients at level 1 are treated as outcomes to be predicted. We have

$$\pi_{pjk} = \beta_{p0k} + \sum_{q=1}^{Q_p} \beta_{pqk} X_{qjk} + r_{pjk} \tag{3}$$

The β_{pqk} are level-2 coefficients, the X_{qjk} level-2 predictors, and r_{pjk} is the level-2 random effect. Taken as a vector, the r's are assumed to have a multivariate normal distribution with a mean vector of $\mathbf{0}$ and a covariance matrix $\boldsymbol{T}_\pi$, with maximum dimension $(P+1) \times (P+1)$.

Level-3 Model

At level 3, the β coefficients at level 2 are treated as outcomes to be predicted. We have

$$\beta_{pqk} = \beta_{pq0} + \sum_{s=1}^{S_{pq}} \gamma_{pqs} W_{sk} + u_{pqk} \tag{4}$$

The γ_{pqs} are level-3 coefficients, the W_{sk} level-2 predictors, and u_{pqk} is the level-3 random effect. Taken as a vector, the u's are assumed to have a multivariate normal distribution with a mean vector of $\mathbf{0}$ and a covariance matrix $\boldsymbol{T}_\beta$, with maximum dimension $\sum_{p=0}^{P}(Q_p+1) \times \sum_{p=0}^{P}(Q_p+1)$.

Introduction

This example, from the HLM6 manual, Chapter 4, has, at level-1, time series data on 1721 students nested within 60 urban public primary schools.

Math achievement is the outcome.

There are 3 SPSS files.

Level-1 File

The file *EG1.SAV* contains the level-1 data. There are two ID variables, `schoolid` for the school, and `childid` for the child. The other variables are

- `year`, the year of the study -3.5
- `grade`, the grade level -1 of the child on each testing occasion
- `math`, a math test score
- `retained`, an indicator for whether the child was retained in grade

Level-2 File

The file *EG2.SAV* contains the level-2 data. It is crucial that the ID's be sorted the same way in both files, with level-2 ID nested within level-3 ID. There are two ID variables, `schoolid` for the school, and `childid` for the child. The other variables are

- `female` (1 = female, 0 = male)
- `black` (1 = African-American, 0 = other)
- `hispanic` (1 = Hispanic, 0 = other)

Level-3 File

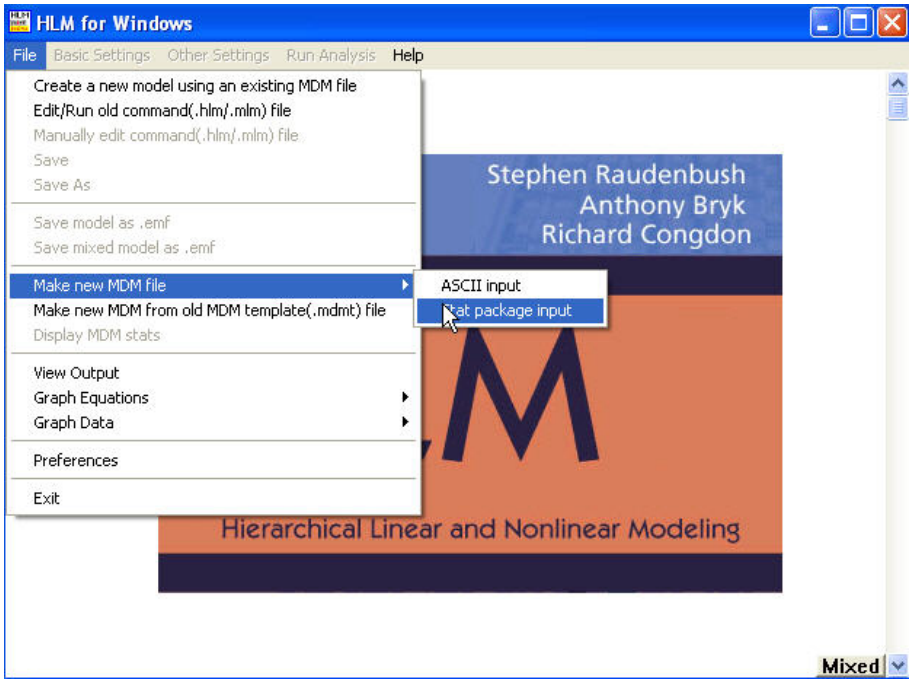
The file *EG3.SAV* contains the level-3 data. The first variable is `schoolid` for the school. The other variables are

- `size`, the number of students enrolled in the school
- `lowinc`, the percentage of students from low income families
- `mobile`, the percentage of students moving during the course of a single academic year

MDM File Setup

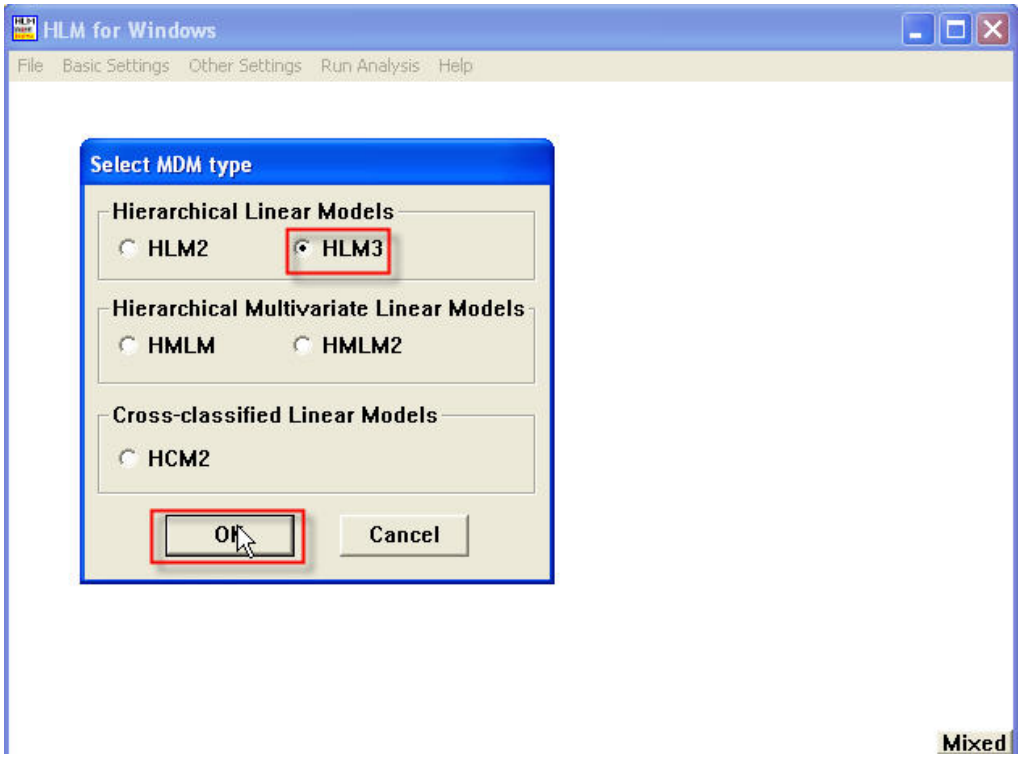
The MDM file setup is very similar to the 2-level setup, except that now variables need to be selected for a third level.

Start up HLM and select “Make new MDM file –> Stat package input”



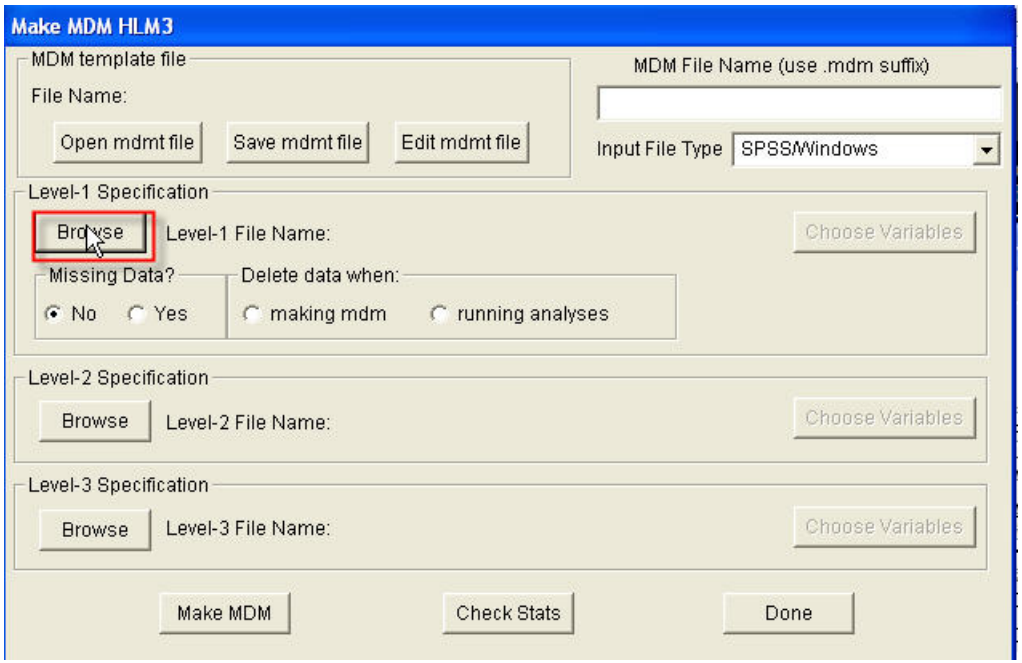
Startup HLM3

Next, select HLM3 and click on OK.



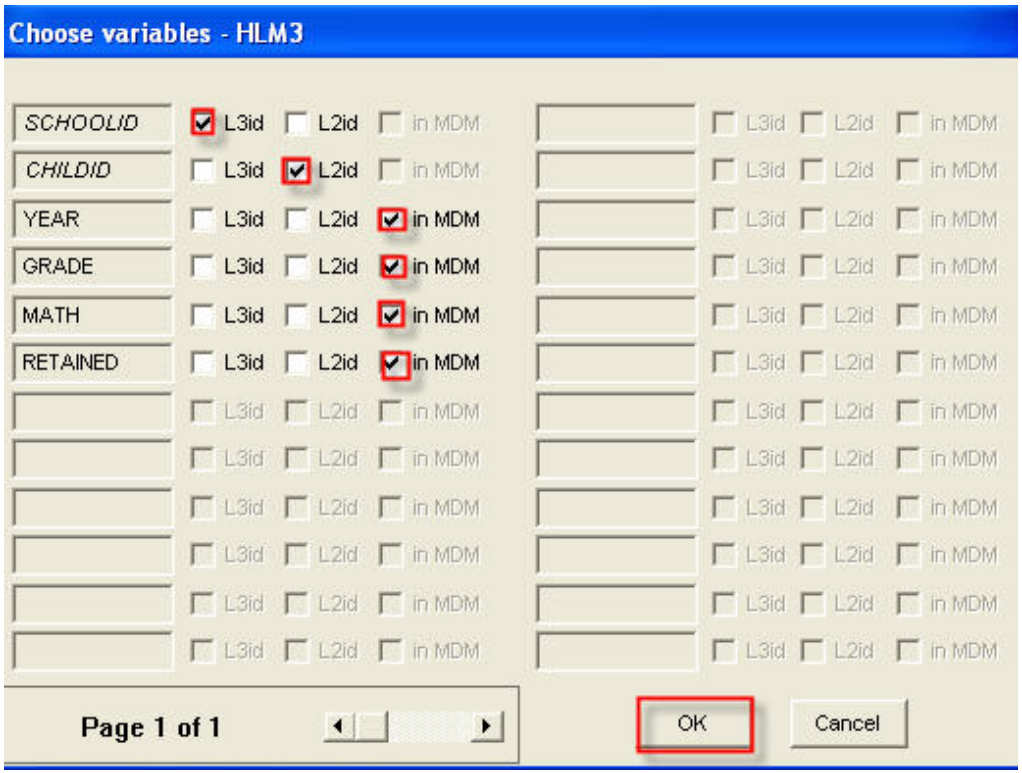
Specify the Variables – Level 1

This will open up the *Make MDM HLM3* specification dialog. Browse for the level-1 specification file, and select *EG1.SAV*. Click *OK*, then click on *Choose Variables*.



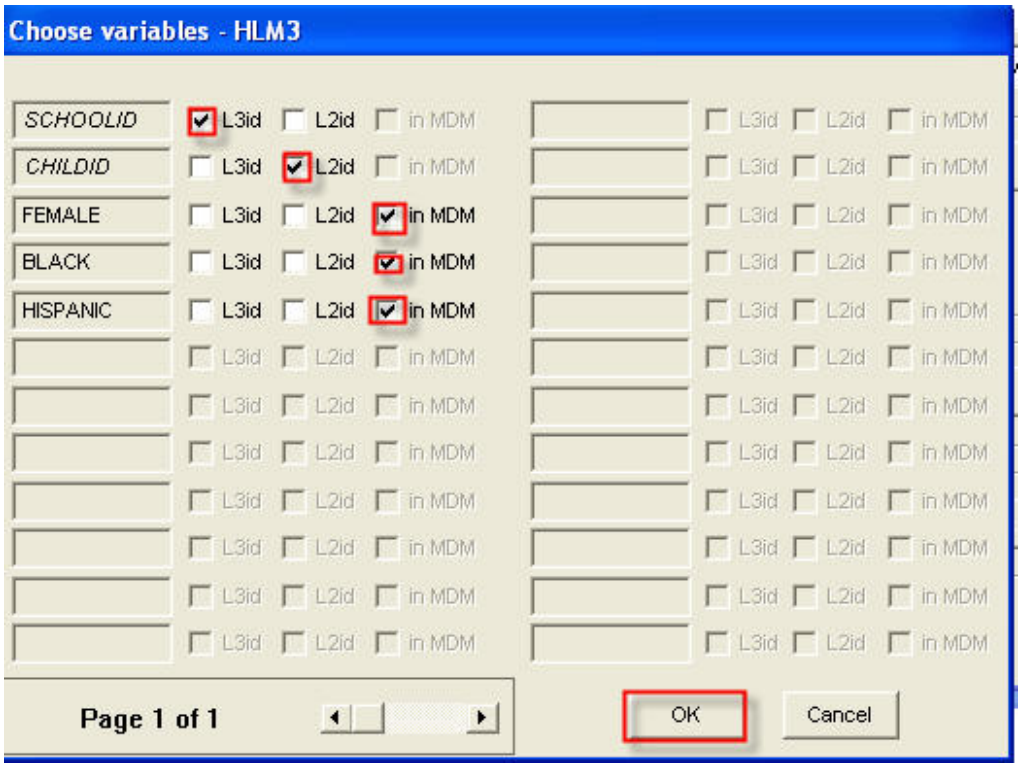
Specify the Variables – Level 1

Select the variables as shown in the snapshot below. This choice should make sense to you!



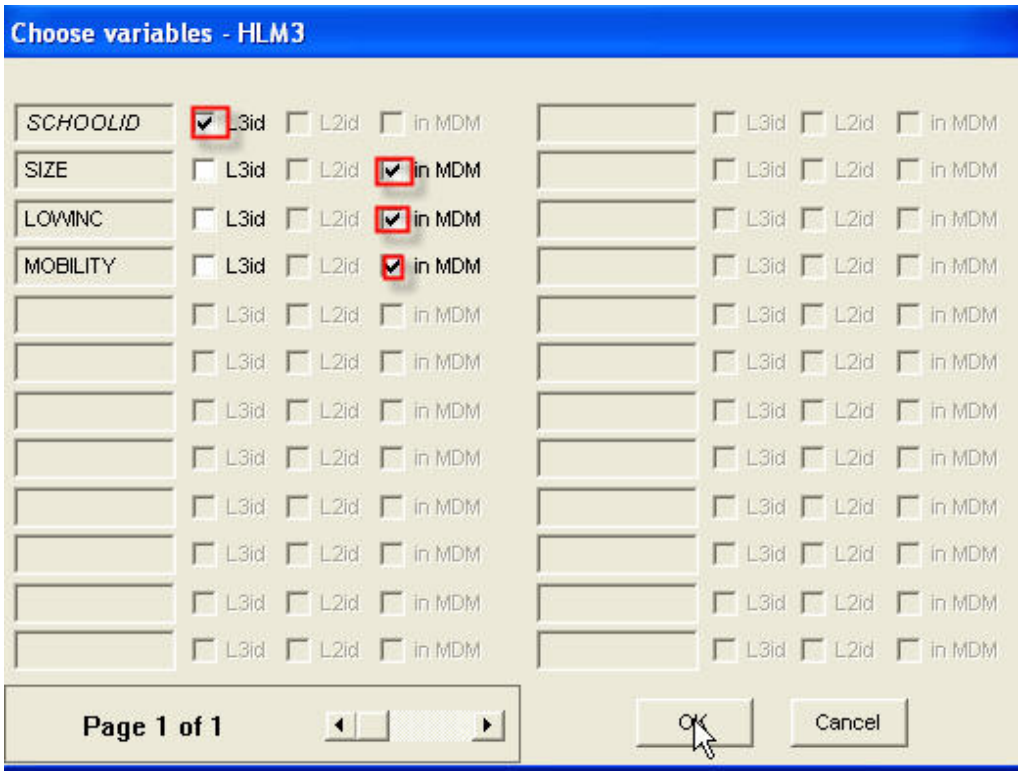
Specify the Variables – Level 2

Browse for a level-2 file. Select *EG2.SAV*. Select the variables as shown in the snapshot below.



Specify the Variables – Level 3

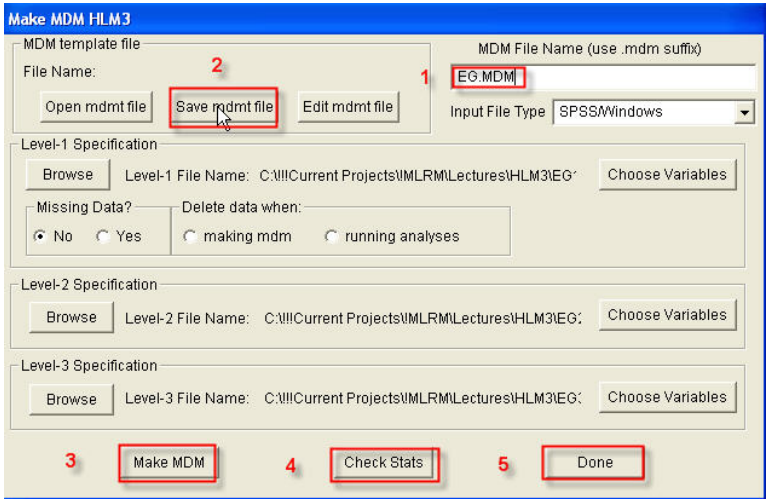
Browse for a level-3 file. Select *EG3.SAV*. The snapshot below shows the proper variable selection.



Create the MDM File

Finish off with these 5 steps, which are marked with red numbers in the screenshot below:

1. Enter the MDM file name in the window.
2. Click *Save mdmt file* and choose *EG* as the file name.
3. Click on *Make MDM file*.
4. Click on *Check Stats*, and make sure they agree with the next slide.
5. Click on *Done*.



Descriptive Statistics

LEVEL-1 DESCRIPTIVE STATISTICS

VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
YEAR	7230	0.38	1.39	-2.50	2.50
GRADE	7230	1.81	1.35	0.00	5.00
MATH	7230	-0.54	1.53	-5.22	5.77
RETAINED	7230	0.05	0.22	0.00	1.00

LEVEL-2 DESCRIPTIVE STATISTICS

VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
FEMALE	1721	0.51	0.50	0.00	1.00
BLACK	1721	0.69	0.46	0.00	1.00
HISPANIC	1721	0.15	0.35	0.00	1.00

LEVEL-3 DESCRIPTIVE STATISTICS

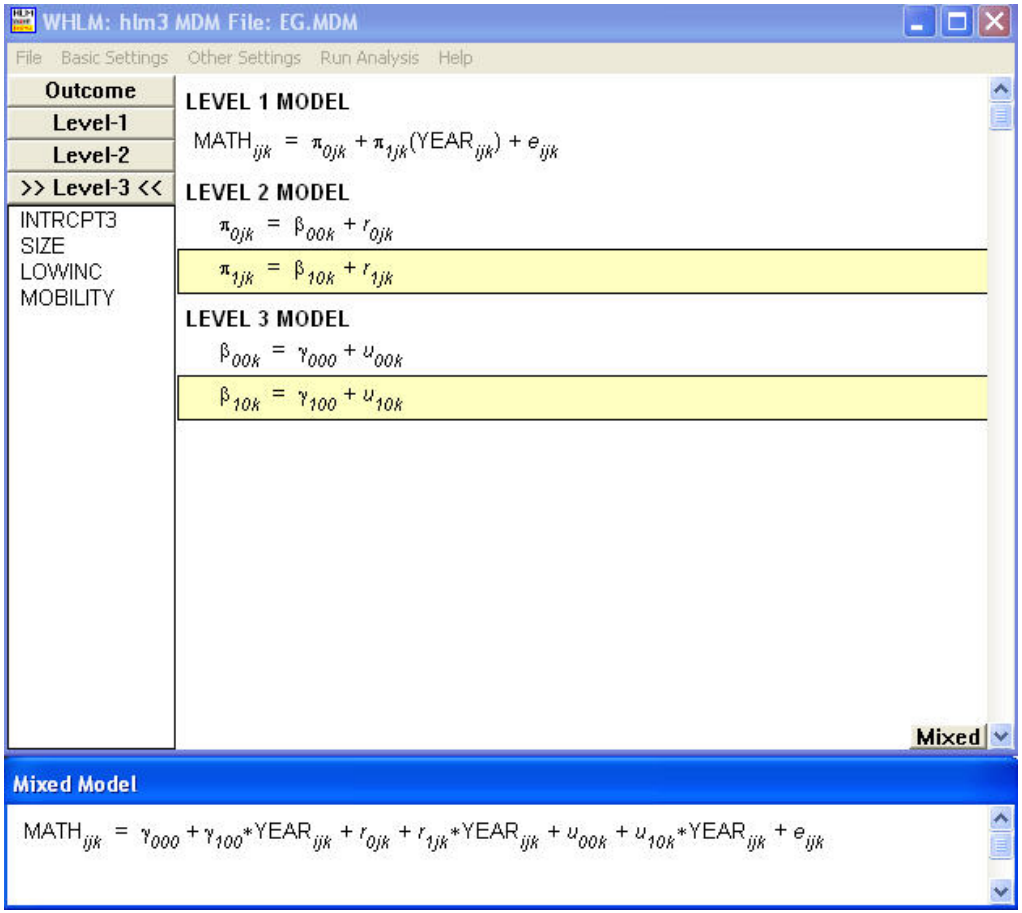
VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
SIZE	60	642.53	317.37	113.00	1486.00
LOWINC	60	73.74	27.27	0.00	100.00
MOBILITY	60	34.75	13.21	8.80	67.00

An Unconditional Growth Model

This model simply states that math achievement (MATH) is predicted linearly by (YEAR). Each child will have a randomly varying slope and intercept, but there are no level-2 or level-3 predictors capturing variation from child or school.

Specifying the Model

Here is the model specification:



A Conditional Model

Here is the model specification: